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Upper and Lower Diagonal Autoregressive Conditional Heteroskedasticity Models as New Classes of March Models

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The primary goal of this paper was to create new classes of models based on the existing Multivariate ARCH models. The MARCH models are used to create Upper and Lower Diagonal models. The models have upper and lower diagonal element parameter restrictions in the coefficient matrices and are found to have the same comparative advantage as MARCH models. Empirical evidence from the Nigerian Urban and Rural Consumer Price Indices has identified UDMARCH and LDMARCH as new classes of models suitable for the volatility of multivariate time series.

Keywords: UDMARCH, LDMARCH, MARCH, Autocorrelations and Cross-Autocorrelations.

1. Introduction

Upper and Lower Diagonal Autoregressive Conditional Heteroskesdaticity Models are subsisting models in the Multivariate Autoregressive Conditional Heteroskedasticity (MARCH) Models identified under certain conditions. MARCH models are multi-response models that provide for interactive effects between the response and predictor variances through their respective lag terms. Upper and Lower Diagonal MARCH models are classes of models whose parameters are restricted to upper and lower diagonal elements of the coefficient matrices. These involve interactions and interdependence amongst the response and predictor conditional variances in each of the Multivariate ARCH models. Bollerslev (1988), Hanson and Hoedahl (1998) introduced Multivariate ARCH models for volatility series with the distributed lags of the squared error as predictor terms. These are generalisation of univariate ARCH models whose conditional variance σ_t^2 is a linear combination of $\epsilon_{t-1}^2, \epsilon_{t-2}^2, \dots \epsilon_{t-q}^2$, Engle et al (1993), Gujarati and Porter (2009). A univariate extension of the ARCH model is the Generalised Autoregressive Conditional Heteroskesdasticity (GARCH). In the GARCH model, the response variance is predicated upon the distributed lag of both the squared error and the variance predicted by $\sigma_{t-1}^2, \sigma_{t-2}^2, \dots \sigma_{t-p}^2$ and $\epsilon_{t-1}^2, \epsilon_{t-2}^2, \dots \epsilon_{t-q}^2$. The model is therefore expressed as a linear combination of autoregressive and moving average processes with "p" and "q" as their respective orders. The multivariate ARCH models involve multi-response conditional variances, such that the lag terms of each are exogenous to other response variances. Given a set of variances σ_{1t}^2, $\sigma_{2t}^2, \dots, \sigma_{nt}^2$ with lag terms of the squared errors $\epsilon_{1t-1}^2, \epsilon_{1t-2}^2, \dots \epsilon_{1t-q}^2; \epsilon_{2t-1}^2, \epsilon_{2t-2}^2, \dots \epsilon_{2t-q}^2$, $\epsilon_{nt-1}^2, \epsilon_{nt-2}^2, \dots \epsilon_{nt-q}^2$ respectively, the models that establish the interactions and interdependence between the response variances and exogenous error terms are Multivariate ARCH models. From the MARCH models, Bollerslev (1988) introduced Pure Diagonal Multivariate ARCH models. The parameters of the models are restricted to only the principal diagonal of the coefficient matrices. Although the models are parsimonious, the parameters restriction to only the principal diagonal in the coefficient matrices does not allow for interdependence amongst the conditional variances. Unlike the upper and lower diagonal matrices of coefficients, which give room for interactions amongst the variances, they provide useful information about the contribution of each predictor term to the response variance. The Upper and Lower Diagonal MARCH Models are parsimonious in their present form, compared to Bollerslev (1988) and Usoro et al (2019). In time series modelling, the adoption of principal of parsimony allows for parameter reduction, therefore, easing the complexity in the estimation of the model parameters. This is important, especially, when the roots of higher order polynomials of the characteristic equation are obtained for stationarity check of the time series process. The Upper and Lower Diagonal MARCH Models proposed in this paper have an advantage over Bollerslev (1988), who introduced Pure Diagonal MARCH as the Bollerslev models took a univariate form having restricted the parameters to only principal diagonal elements in the coefficient matrices. By implication, Bollerslev (1988) do not give room for interactive effects between each response and lag terms of predictive variables. Notwithstanding the parameter restrictions to upper and lower diagonal elements of the coefficient matrices in this paper, the new classes of MARCH Models do not negate the feedforward and feedback mechanism between each response and the lag terms of other predictive variables. The fact is that, increase in the number predictive variables in an estimated models maximises the chances of multicollinearity amongst the lagged explanatory variables. The introduction of the Upper and Lower Diagonal MARCH Models which are limited to upper and lower diagonal elements of the parameter matrices partly addresses the issues of multicollinearity in the regression estimates of the multivariate time series models. The major focus of this work is to propose upper and lower diagonal multivariate autoregressive conditional heteroskedasticity models as new classes of MARCH models which in effect, avert the complexity in the estimation of model parameters, as well as minimizes the chances of multicollinearity in the estimated models.

2. Review

This section deals with the review of related works on multivariate volatility models. The multivariate analogous to the ARCH (q) model is the MARCH (q) of multiple response variances by Bollerslev et al (1988), Engle and Kroner (1995) and Luc et al (2006). Bollerslev et al (1988) also introduced the DVECH model for K=2, restricting the parameters of the coefficient matrices to pure diagonal elements. These models are parsimonious with the parameter restrictions. The proposed models are Diagonal Autoregressive Conditional Heteroskedasticity Models. DVECH by Bollerslev et al (1988) has associated parameters of the first and second lag terms of ϵ_t^2, precluding the interactions and interdependence amongst the predictor and response variances in the Multivariate ARCH models. These are pure diagonal ARCH models. On Lower diagonal model included Usoro and Omekara (2007). Other diagonal time series models included Covariance analysis of the squares of the purely diagonal bilinear time series models by Iwueze and Johnson (2011), and the properties of pure diagonal bilinear models by Omekara (2016). In this paper, we propose Upper and Lower Diagonal Autoregressive Conditional Heteroskedasticity Models from the existing MARCH models. What differentiates the proposed Upper and Lower Diagonal MARCH models from the DVECH models introduced by Bollerslev et al (1988) is that the earlier contain interactive parameters of the response and predictor variables in the matrix of coefficients, while the latter models have parameter restrictions to pure diagonal elements, which takes a univariate form.

Usoro et al (2019) fitted MARCH models to Nigeria Urban, Rural and Average Consumer Price Indices. The models were not parsimonious in their form because of the multiple parameter estimates. In this paper, data on consumer price indices will be used to compare the performances between the general MACH Models and Upper/Lower Diagonal MARCH Models. As part of the properties of time series models, this paper also considers Upper and Lower Autocorrelations and Cross-Autocorrelations for the MARCH models.

3. Methodology

3.1 Diagonal MARCH Models from Multivariate ARCH Models

PROPOSITION

Let $X_{it(i=1,\dots,m)}$ be a set of multivariate time processes with variances $\sigma^2_{it(i=1,\dots,m)}$, squared error terms $\epsilon^2_{vt(v=1,\dots,n)}$ and constants $\gamma_{i(i=1,\dots,m)}$, ϵ^2_{vt-s} are lag terms of the squared error such that $\sigma^2_{it(i=1,\dots,m)}$ are functions of ϵ^2_{vt-s} with respective matrices of parameters $\theta_{iv.s(v=1,\dots,n)}$. If the number of $\theta_{1v.s} > \theta_{2v.s} > \cdots > \theta_{mv.s}$, then we have Upper Diagonal Autoregressive Conditional Heteroskedasticity (UDMARCH) Models. If the number of $\theta_{1v.s} < \theta_{2v.s} < \cdots < \theta_{mv.s}$, then we have Lower Diagonal Autoregressive Conditional Heteroskedasticity (LDMARCH).

Derivation:

From the above proposition, let a set of Multivariate ARCH Models by Usoro et al (2019) be presented as follows

$$\sigma^2_{it} = \gamma_i + \sum_{s=1}^{q}\sum_{v=1}^{n} \theta_{iv.s}\epsilon^2_{vt-s}, i = 1, \dots, m \quad (1)$$

Case 1: Upper Diagonal MARCH Models

Given σ^2_{it}; $if\ i = 1;\ v = 1, 2, 3, \dots, n; s = 1, 2, \dots, q$

$if\ i = 2;\ v = 2, 3, \dots, n; s = 1, 2, \dots, q$

$if\ i = 3;\ v = 3, \dots, n; s = 1, 2, \dots, q$

$if\ i = m;\ v = n; s = 1, 2, \dots, q$

σ_{it}^2 has a set of models with the number of sequential coefficients $\theta_{1v.s} > \theta_{2v.s} > \cdots > \theta_{mv.s}$, which describe the upper diagonal parameter matrices. Hence, the Upper Diagonal MARCH model is presented as

$$\sigma_{it}^2 = \begin{cases} \gamma_1 + \theta_{1v.s}\epsilon_{vt-s}^2, v = 1,2,3,\dots,n; s = 1,2,\dots,q \\ \gamma_2 + \theta_{2v.s}\epsilon_{vt-s}^2, v = 2,3,\dots,n; s = 1,2,\dots,q \\ \gamma_3 + \theta_{3v.s}\epsilon_{vt-s}^2, v = 3,\dots,n; s = 1,2,\dots,q \\ \vdots \\ \gamma_m + \theta_{mv.s}\epsilon_{vt-s}^2, v = n; s = 1,2,\dots,q \end{cases}, for\ \theta_{1v.s} > \theta_{2v.s} > \cdots > \theta_{mv.s} \quad (2)$$

Proof:

Case 1:

From Equation (1), we have

$$\sigma_{it}^2 = \gamma_i + \sum_{v=1}^{n}\sum_{s=1}^{q}\theta_{iv.s}\epsilon_{vt-s}^2, i = 1,\dots,m$$

Expanding the above model gives

$$\sigma_{it}^2 = \gamma_i + \sum_{v=1}^{n}\left[\theta_{iv.1}\epsilon_{vt-1}^2 + \theta_{iv.2}\epsilon_{vt-2}^2 + \cdots + \theta_{iv.q}\epsilon_{vt-p}^2\right], i = 1,\dots,m$$

$$= \gamma_i + \left[(\theta_{i1.1}\epsilon_{1t-1}^2 + \theta_{i2.2}\epsilon_{2t-1}^2 + \cdots + \theta_{in.1}\epsilon_{nt-1}^2) + (\theta_{i1.2}\epsilon_{1t-2}^2 + \theta_{i2.2}\epsilon_{2t-2}^2 + \cdots + \theta_{in.2}\epsilon_{nt-2}^2) + \cdots + \left(\theta_{i1.q}\epsilon_{1t-q}^2 + \theta_{i2.q}\epsilon_{2t-q}^2 + \cdots + \theta_{in.q}\epsilon_{nt-q}^2\right)\right], i = 1,\dots,m \quad (3)$$

From Equation (3), for $i = 1; v = 1,2,\dots,n; s = 1,2,\dots,q$, we have

$$\sigma_{1t}^2 = \gamma_1 + \theta_{11.1}\epsilon_{1t-1}^2 + \theta_{12.1}\epsilon_{2t-1}^2 + \cdots + \theta_{1n.1}\epsilon_{nt-1}^2 + \theta_{11.2}\epsilon_{1t-2}^2 + \theta_{12.2}\epsilon_{2t-2}^2 + \cdots + \theta_{1n.2}\epsilon_{nt-2}^2 + \theta_{11.q}\epsilon_{1t-q}^2 + \theta_{12.q}\epsilon_{2t-q}^2 + \cdots + \theta_{1n.q}\epsilon_{nt-q}^2 \quad (4)$$

From Equation (4),

$$\sigma_{1t}^2 = \gamma_1 + \theta_{1v.s}\epsilon_{vt-s}^2, v = 1,2,3,\dots,n; s = 1,2,\dots,q \quad (5)$$

For $i = 2; v = 2,3,\dots,n; s = 1,2,\dots,q$, we have,

$$\sigma_{2t}^2 = \gamma_2 + \theta_{22.1}\epsilon_{2t-1}^2 + \theta_{23.1}\epsilon_{3t-1}^2 + \cdots + \theta_{2n.1}\epsilon_{nt-1}^2 + \theta_{22.2}\epsilon_{2t-2}^2 + \theta_{23.2}\epsilon_{3t-2}^2 + \cdots + \theta_{2n.2}\epsilon_{nt-2}^2 + \theta_{22.q}\epsilon_{2t-q}^2 + \theta_{23.q}\epsilon_{3t-q}^2 + \cdots + \theta_{2n.q}\epsilon_{nt-q}^2 \quad (6)$$

From Equation (6),

$$\sigma_{2t}^2 = \gamma_2 + \theta_{2v.s}\epsilon_{vt-s}^2, v = 2,3,\dots,n; s = 1,2,\dots,q \quad (7)$$

For $i = 3; v = 3,4,\dots,n; s = 1,2,\dots,q$, we have,

$$\sigma_{3t}^2 = \gamma_3 + \theta_{33.1}\epsilon_{3t-1}^2 + \theta_{34.1}\epsilon_{4t-1}^2 + \cdots + \theta_{3n.1}\epsilon_{nt-1}^2 + \theta_{33.2}\epsilon_{3t-2}^2 + \theta_{34.2}\epsilon_{4t-2}^2 + \cdots + \theta_{3n.2}\epsilon_{nt-2}^2 + \theta_{33.q}\epsilon_{3t-q}^2 + \theta_{34.q}\epsilon_{4t-q}^2 + \cdots + \theta_{3n.q}\epsilon_{nt-q}^2 \quad (8)$$

From Equation (8),

$$\sigma_{3t}^2 = \gamma_3 + \theta_{3v.s}\epsilon_{vt-s}^2, v = 3,4,\dots,n; s = 1,2,\dots,q \quad (9)$$

For $i = m; v = n; s = 1,2,\dots,q$, we have,

$$\sigma_{mt}^2 = \gamma_m + \theta_{mn.1}\epsilon_{nt-1}^2 + \theta_{mn.2}\epsilon_{nt-2}^2 + \cdots + \theta_{mn.q}\epsilon_{nt-q}^2 \quad (10)$$

From Equation (10),

$$\sigma_{mt}^2 = \gamma_m + \theta_{mv.s}\epsilon_{vt-s}^2, v = n; s = 1,2,\dots,q \quad (11)$$

Therefore, Equations (5), (7), (9) and (11) are a compendium of UDMARCH models, which complete the proof.

Case 2: Lower Diagonal MARCH Models

Given $\sigma_{it}^2;\ if\ i = 1;\ v = 1; s = 1, 2, \dots, q$

$$if\ i = 2;\ v = 1, 2; s = 1, 2, \dots, q$$
$$if\ i = 3;\ v = 1, 2, 3; s = 1, 2, \dots, q$$
$$if\ i = m;\ v = 1, 2, 3, \dots, n; s = 1, 2, \dots, q$$

σ_{it}^2 has a set of models with the number of sequential coefficients $\theta_{1v.s} < \theta_{2v.s} < \cdots < \theta_{mv.s}$, which describe the lower diagonal parameter matrices. Hence, the Lower Diagonal MARCH model is presented as

$$\sigma_{it}^2 = \begin{cases} \gamma_1 + \theta_{1v.s}\epsilon_{vt-s}^2, v = 1; s = 1,2,\dots,q \\ \gamma_2 + \theta_{2v.s}\epsilon_{vt-s}^2, v = 1,2; s = 1,2,\dots,q \\ \gamma_3 + \theta_{3v.s}\epsilon_{vt-s}^2, v = 1,2,3; s = 1,2,\dots,q \\ \vdots \\ \gamma_m + \theta_{mv.s}\epsilon_{vt-s}^2, v = 1,2,3,\dots,n; s = 1,2,\dots,q \end{cases} , for\ \theta_{1v.s} < \theta_{2v.s} < \cdots < \theta_{mv.s} \quad (12)$$

Proof:

From Equation (3), the necessary conditions are set. Thus,

For $i = 1;\ v = 1; s = 1,2,\dots,q$, we have

$$\sigma_{1t}^2 = \gamma_1 + \theta_{11.1}\epsilon_{1t-1}^2 + \theta_{11.2}\epsilon_{1t-2}^2 + \theta_{11.q}\epsilon_{1t-q}^2 \quad (13)$$

From Equation (13),

$$\sigma_{1t}^2 = \gamma_1 + \theta_{1v.s}\epsilon_{vt-s}^2, v = 1; s = 1,2,\dots,q \quad (14)$$

For $i = 2;\ v = 1, 2; s = 1,2, \dots, q$, we have

$$\sigma_{2t}^2 = \gamma_2 + \theta_{21.1}\epsilon_{1t-1}^2 + \theta_{22.1}\epsilon_{2t-1}^2 + \theta_{21.2}\epsilon_{1t-2}^2 + \theta_{22.2}\epsilon_{2t-2}^2 + \cdots + \theta_{21.q}\epsilon_{1t-q}^2 + \theta_{22.q}\epsilon_{2t-q}^2 \quad (15)$$

From Equation (15),

$$\sigma_{2t}^2 = \gamma_2 + \theta_{2v.s}\epsilon_{vt-s}^2, v = 1, 2; s = 1,2, \dots, q \quad (16)$$

For $i = 3; v = 1,2\ 3; s = 1,2, \dots, q$, we have

$$\sigma_{3t}^2 = \gamma_3 + \theta_{31.1}\epsilon_{1t-1}^2 + \theta_{32.1}\epsilon_{2t-1}^2 + \theta_{33.1}\epsilon_{3t-1}^2 + \theta_{31.2}\epsilon_{1t-2}^2 + \theta_{32.2}\epsilon_{2t-2}^2 + \theta_{33.2}\epsilon_{3t-2}^2 + \cdots + \theta_{31.q}\epsilon_{1t-q}^2 + \theta_{32.q}\epsilon_{2t-q}^2 + \theta_{33.q}\epsilon_{3t-q}^2 \quad (17)$$

From Equation (17),

$$\sigma_{3t}^2 = \gamma_3 + \theta_{3v.s}\epsilon_{vt-s,}^2\ v = 1,2\ 3; s = 1,2, \dots, q \quad (18)$$

For $i = m; v = 1, 2, \dots, n; s = 1,2, \dots, q$, we have,

$$\sigma_{mt}^2 = \gamma_m + \theta_{m1.1}\epsilon_{1t-1}^2 + \theta_{m2.1}\epsilon_{2t-1}^2 + \cdots + \theta_{mn.1}\epsilon_{nt-1}^2 + \theta_{m1.2}\epsilon_{1t-2}^2 + \theta_{m2.2}\epsilon_{2t-2}^2 + \cdots + \theta_{mn.2}\epsilon_{nt-2}^2 + \dots + \theta_{m1.q}\epsilon_{1t-q}^2 + \theta_{m2.q}\epsilon_{2t-q}^2 + \theta_{mn.q}\epsilon_{nt-q}^2 \quad (19)$$

From Equation (19),

$$\sigma_{mt}^2 = \gamma_m + \varphi_{mj.k}\sigma_{jt-k}^2 + \theta_{mv.s}\epsilon_{vt-s}^2, v = 1, 2, \dots, n; s = 1,2, \dots, q \quad (20)$$

Therefore, Equations (14), (16), (18) and (20) are a set of LDMARCH models, for $\varphi_{1j.k} < \varphi_{2j.k} < \cdots < \varphi_{mj.k}\ and\ \theta_{1v.s} < \theta_{2v.s} < \cdots < \theta_{mv.s}$. These complete the proof.

What differentiate the two models are the parameter restrictions to the upper and lower diagonal elements of the coefficient matrices. The restriction of the parameters of the new classes of MARCH models is to avert heavy parameterization in the models. This guarantees parsimonious Multivariate ARCH. As parsimonious models, the UDMARCH and LDMARCH models modify Bollerslev (1988) and Usoro et al (2019).

3.2 Autocovariances and Cross-Autocovariances

The Autocovariances and Cross-Autocovariances of the multivariate volatility series are presented as

$$\tau_{it+k,jt+l} = \begin{bmatrix} \tau_{1t+k,1t+l} & \tau_{1t+k,2t+l} & \tau_{1t+k,3t+l} \cdots & \tau_{1t+k,nt+l} \\ \tau_{2t+k,1t+l} & \tau_{2t+k,2t+l} & \tau_{2t+k,3t+l} \cdots & \tau_{2t+k,nt+l} \\ \tau_{3t+k,1t+l} & \tau_{3t+k,2t+l} & \tau_{3t+k,3t+l} \cdots & \tau_{3t+k,nt+l} \\ \vdots & \vdots & \vdots \quad \cdots & \vdots \\ \tau_{mt+k,1t+l} & \tau_{mt+k,2t+l} & \tau_{mt+k,3t+l} \cdots & \tau_{mt+k,nt+l} \end{bmatrix} \quad (21)$$

Each $\tau_{it+k,jt+l}$ in (21) is a sub-matrix of autocovariance or cross-autocovariance of the processes.

3.3 Autocorrelations and Cross-Autocorrelations

Autocorrelations measure the correlation between each response time variable and its lag terms, while. It is the correlation between $X_t \, and \, X_{t-k}$. Cross-Autocorrelations measure the correlations between two different time variables and their lag terms; correlation $X_{it} \, and \, X_{jt-k}$ or$X_{jt} \, and \, X_{it-k} (i \neq j)$. The Autocorrelations and Cross-Autocorrelations of the multivariate volatility series are presented here;

$$\rho_{it+k,jt+l} = \begin{bmatrix} \rho_{1t+k,1t+l} & \rho_{1t+k,2t+l} & \rho_{1t+k,3t+l} \cdots & \rho_{1t+k,nt+l} \\ \rho_{2t+k,1t+l} & \rho_{2t+k,2t+l} & \rho_{2t+k,3t+l} \cdots & \rho_{2t+k,nt+l} \\ \rho_{3t+k,1t+l} & \rho_{3t+k,2t+l} & \rho_{3t+k,3t+l} \cdots & \rho_{3t+k,nt+l} \\ \vdots & \vdots & \vdots \quad \cdots & \vdots \\ \rho_{mt+k,1t+l} & \rho_{mt+k,2t+l} & \rho_{mt+k,3t+l} \cdots & \rho_{mt+k,nt+l} \end{bmatrix} \quad (22)$$

where, $\rho_{it+k,jt+l} = \frac{\gamma_{it+k,jt+l}}{\sqrt{\gamma_{it}\gamma_{jt}}}$ and $i = 1, \ldots, m; j = 1, \ldots, n; k = 0, \ldots, r; l = 0, \ldots, s$

The autocorrelation and cross-autocorrelation matrix of the response and predictor variables is symmetrical. Therefore, the elements in the upper and lower diagonal for the response and predictor variances remain the same. The autocorrelation is the correlation between each response variance and its respective lag term. The cross-autocorrelation is the correlation between a conditional variance and the lag term of the other conditional variance Usoro (2020). This paper also presents the diagonal structure of autocorrelations and cross-autocorrelations as properties of upper and lower diagonal multivariate autoregressive conditional heteroskedasticity models.

Case 1: Identification of Upper Diagonal Autocorrelation and Cross-Autocorrelation Matrix

From Equation (22), $\rho_{it+k,jt+l(i=1,\ldots,m;j=1,\ldots,n;k=0,1,\ldots,,r;l=0,1,\ldots,s)}$ is the general autocorrelation and cross-autocorrelation of response and predictor conditional variance. We present the upper diagonal autocorrelation and cross-autocorrelation functions as,

$$\rho_{it+k,jt+l} = \begin{cases} \rho_{1t+k,jt+l}, & i = 1, j = 2,3,4,\ldots,n \\ \rho_{2t+k,jt+l}, & i = 2, j = 2,3,4,\ldots,n \\ \rho_{3t+k,jt+l}, & i = 3, j = 2,3,4,\ldots,n \\ \vdots & \\ \rho_{mt+k,jt+l}, & i = m, j = n(m = n) \end{cases} \tag{23}$$

Case 2: Identification of Lower Diagonal Autocorrelation and Cross-Autocorrelation Matrix

From Equation (22), $\rho_{it+k,jt+l(i=1,\ldots,m;j=1,\ldots,n;k=0,1,\ldots,,r;l=0,1,\ldots,s)}$ present the lower diagonal autocorrelation and cross-autocorrelation functions as,

$$\rho_{it+k,jt+l} = \begin{cases} \rho_{1t+k,jt+l}, & i = 1, j = 1 \\ \rho_{2t+k,jt+l}, & i = 2, j = 1,2 \\ \rho_{3t+k,jt+l}, & i = 3, j = 1,2,3 \\ \vdots & \\ \rho_{mt+k,jt+l}, & i = m, j = 1,2,3,\ldots,n(m = n) \end{cases} \tag{24}$$

Equations (23) and (24) represent the upper and lower diagonal autocorrelation and cross-autocorrelation matrices respectively. The two triangular matrices are symmetrical. The autocorrelations and cross-autocorrelations are very useful in the choice of the model for each volatility series. The lag length can be determined through the usual ACF and PACF as applicable in the autoregressive and moving average processes.

3.4 Data Source and Volatility Measure

The data for the work are Nigerian Urban and Rural Consumer Prices Indices collected from the Website of the Central Bank of Nigeria. The range is from November 2009 to February 2016, with November 2009 as the constant basic price. The variance σ_t^2 is a volatility measure, which is obtained as the square of the return series of each consumer price index. Thus,

$$\sigma_t = \frac{\epsilon_t}{Z_t} => \sigma_t^2 = \frac{\epsilon_t^2}{Z_t^2} \tag{25}$$

where, σ_t, ϵ_t and Z_t represent the return series, error term from the model and standard normal random variable. $\epsilon_t \sim N(0, \sigma_e^2)$ and $Z_t \sim N(0,1)$. The error can be obtained as the product of the return series and standard normal random variable simulated with zero mean and unit variance. Alternatively, the error can be obtained as the residual of the preliminary regression estimates of the stationary series. The square of the error is the predictive time variable, whose lag terms are the predetermining factors to the volatility response time variable.

4. Estimation of Models

This section presents estimates of model parameters for the two series on the basis of the distributions of the autocorrelation and partial autocorrelation functions. Although, the plots of ACF and PACF of the volatility series are not shown, but form the basis for the choice and order of each model.

Table1: Estimation of Parameters of the MARCH [P(3, 0)] Models

Predictor	Coefficients	SE. Coefficients	T	P
Urban CPI				
ϵ^2_{1t-1}	0.1563	0.0922	1.69	0.093
ϵ^2_{1t-2}	-0.0053	0.0892	-0.06	0.952
ϵ^2_{1t-3}	0.0435	0.0865	0.50	0.615
ϵ^2_{2t-1}	-0.0833	0.0510	-1.63	0.105
ϵ^2_{2t-2}	0.1074	0.0429	2.50	0.014
ϵ^2_{2t-3}	0.0714	0.0522	1.37	0.174
Rural CPI				
ϵ^2_{1t-1}	0.145	0.163	0.88	0.378
ϵ^2_{1t-2}	-0.064	0.158	-0.41	0.686
ϵ^2_{1t-3}	0.323	0.153	2.11	0.037
ϵ^2_{2t-1}	0.2198	0.0903	2.43	0.016
ϵ^2_{2t-2}	0.4916	0.0760	6.47	0.000
ϵ^2_{2t-3}	0.1764	0.0925	1.91	0.059

Table2: Estimates of Parameters of the UDMARCH [P(3,0)] Models

Predictor	Coefficients	SE. Coefficients	T	P
Urban CPI Estimates				
ϵ^2_{1t-1}	0.1563	0.0922	1.69	0.093
ϵ^2_{2t-1}	-0.0833	0.0510	-1.63	0.105
ϵ^2_{1t-2}	-0.0053	0.0892	-0.06	0.952
ϵ^2_{2t-2}	0.1074	0.0429	2.50	0.014
ϵ^2_{1t-3}	0.0435	0.0865	0.50	0.615
ϵ^2_{2t-3}	0.0714	0.0522	1.37	0.174
Rural CPI Estimates				
ϵ^2_{1t-1}	0.2027	0.0870	2.33	0.021
ϵ^2_{1t-2}	0.5220	0.0752	6.94	0.000
ϵ^2_{1t-3}	0.1964	0.0864	2.27	0.025

Table3: Estimates of Parameters of the LDMARCH [P(3,0)] Models

Predictor	Coefficients	SE. Coefficients	T	P
Urban CPI Estimates				
ϵ^2_{1t-1}	0.2638	0.0882	2,99	0.003
ϵ^2_{1t-2}	-0.0124	0.0912	-0.14	0.892
ϵ^2_{1t-3}	0.1100	0.0882	1.25	0.215
Rural CPI Estimates				
ϵ^2_{1t-1}	0.145	0.163	0.88	0.378
ϵ^2_{2t-1}	0.2198	0.0903	2.43	0.016
ϵ^2_{1t-2}	-0.64	0.158	-0.41	0.686
ϵ^2_{2t-2}	0.4916	0.0760	6.47	0.000
ϵ^2_{1t-3}	0.323	0.153	2.11	0.037
ϵ^2_{2t-3}	0.1764	0.0925	1.91	0.059

Table 4: Descriptive Statistics for the Errors of MARCH and DMARCH Models

Variable	N	Mean	SE. Mean	St. Dev	Sum of Squares
MARCH (σ_{1t}^2)	130	0.00000	0.000003	0.000035	0.000000
MARCH (σ_{2t}^2)	130	0.00000	0.000005	0.000055	0.000000
UDMARCH (σ_{1t}^2)	130	0.00000	0.000003	0.000036	0.000000
UDMARCH (σ_{2t}^2)	130	0.00000	0.000005	0.000056	0.000000
LDMARCH (σ_{1t}^2)	130	0.00000	0.000003	0.000036	0.000000
LDMARCH (σ_{2t}^2)	130	0.00000	0.000005	0.000055	0.000000

Tables 1, 2 and 3 present parameter estimates for the MARCH, UDMARCH and LDMARCH models. Table 4 presents the basics statistics showing the performances of the three models. The basic statistics of the errors in the table for the three models reveals zero mean with values of the standard deviation. These showcase equal comparative advantage of Bollerslev (1988) and Usoro et al (2019). The three models capture volatility clustering in the Urban and Rural Nigeria Consumer Price Indices. What is very interesting in the Upper and Lower Diagonal MARCH models is that restrictions in the parameters of the models to the upper and lower diagonals of the coefficient matrices does not negatively affect the estimated model as evident in the basic statistics in Table 4. Besides the equality in the performances between Bollerslev's and these new classes of multivariate volatility models, Upper and Lower Diagonal MARCH models are parsimonious with the reduction in the number of parameters in the models. This is a novel idea and important aspect of this work.

Summary and Conclusion

The interest in this paper was motivated by the need to identify classes of diagonal models from the existing MARCH models for volatility series. This ascertained stability in the volatility series. MARCH models are multivariate ARCH models with interactive effects of the predictor and response lag terms on each response conditional variance. In this paper, we have introduced Upper and Lower Diagonal MARCH models which are linear combinations of the lag terms of the squared error. The new set of models have a reduction in the number of parameters from the general MARCH models with interactions and interdependence on each response variance. Advantageously, the UDMARCH and LDMARCH models are parsimonious with parameter restrictions to only upper and lower diagonal elements in the coefficient matrices. Besides, the basic statistics in Table 4 is an indication of equal performances of the new set of MARCH models and the existing ones. Apart from the principle of parsimony observed in the new models through reduction in the model parameters, another important aspect of Upper and Lower Diagonal MARCH models is the guard against presence of multicollinearity amongst the predictive lag terms. As much as the new classes of MARCH models have equal performance with the existing ones, it connotes partial or full elimination of some lag terms whose parameters may not be significant in each of the models. This parameter restriction minimizes the number of predictive time variables in a multiple linear relationship. Hence, these models are established as new classes of MARCH models for multivariate volatility series.

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